

An Introduction to  
**Latent Variable  
Growth Curve  
Modeling**

**Concepts, Issues,  
and Applications**

*Second Edition*

**Terry E. Duncan**  
**Susan C. Duncan**  
**Lisa A. Strycker**  
*Oregon Research Institute*



LAWRENCE ERLBAUM ASSOCIATES, PUBLISHERS  
Mahwah, New Jersey London

# Contents

Preface	ix
Acknowledgments	xii
1. Introduction	1
Typical Approaches to Studying Change	1
Toward an Integrated Developmental Model	3
Organization of the Book	5
Related Literature on LGM	9
Software Implementation	10
Evaluation of Model Fit	13
2. Specification of the LGM	17
Two-Factor LGM for Two Time Points	17
LGM Parameters	19
LGM Assumptions	21
Expressing Model Parameters as Functions of Measured Means, Variances, and Covariances	21
Interpretation of the Growth Factors	23
Representing the Shape of Growth Over Time	26
Example 2.1: Three-Factor Polynomial LGM	26
Example 2.2: Unspecified Two-Factor LGM	31
Example 2.3: The Single-Factor LGM	35
Summary	38
3. LGM, Repeated Measures ANOVA, and the Mixed Linear Model	41
Example 3.1: The Unconditional Growth Curve Model	42
Including Predictors and Sequelae of Change in Growth Curve Models	50

Example 3.2: Growth Curve Models Involving Predictors of Change	52
Example 3.3: Growth Curve Models Involving Sequelae of Change	56
Example 3.4: The Full Growth Curve Model Involving Predictors and Sequelae of Change	58
Summary	61
4. Multivariate Representations of Growth and Development	63
Example 4.1: Associative LGM	64
Higher Order LGMs	67
Example 4.2: Factor-of-Curves LGM	68
Example 4.3: Curve-of-Factors LGM	69
Example 4.4: Including Structural Parameters	74
Summary	77
5. Analyzing Growth in Multiple Populations	81
Equality of Sets of Parameters of an LGM	83
Example 5.1: Multiple-Sample Analysis of Change	84
Lagrange Multipliers	86
Example 5.2: Alternative Multiple-Sample Analysis of "Added Growth" LGM	88
Summary	90
6. Accelerated Designs	93
Cohort-Sequential LGM	94
Example 6.1: Cohort-Sequential LGM	97
Example 6.2: Unspecified Cohort-Sequential LGM	98
Summary	100
7. Multilevel Longitudinal Approaches	103
Example 7.1: Full Information Maximum Likelihood Estimation (FIML)	105
Example 7.2: Multilevel LGM (MLGM)	109
Example 7.3: Extension of the Hierarchical LGM to Four Levels	115
Summary	122

---

8. Growth Mixture Modeling	125
Latent Class Analysis of Dynamic Models	125
Covariance Structure Analysis Mixture Modeling	126
Growth Mixture Modeling	127
Model Specifications	128
Model Estimation	131
Example 8.1: The Single-Class Growth Curve Model	132
Example 8.2: Determining Sample Heterogeneity: Multiple-Class Growth Curve Models	135
Alternative Methods for Estimating the Number of Classes and Parameter Starting Values	138
Example 8.3: Including Covariates in the Mixture Modeling Framework	141
Example 8.4: Including Mixture Indicators	143
Summary	147
9. Piecewise and Pooled Interrupted Time Series LGMs	151
Example 9.1: Piecewise Models	153
Example 9.2: Pooled Interrupted Time Series LGM	157
Example 9.3: Simple Change LGM	160
Summary	162
10. Latent Growth Curve Modeling With Categorical Variables	165
Measurement Characteristics of the Ordered Categorical Variable	167
Growth Modeling With Categorical Outcome Variables	168
Software Implementation	169
Example 10.1: LGM of Ordered Categorical Outcomes	172
Summary	176
11. Missing Data Models	179
A Taxonomy of Methods for Partial Missingness	179
A Taxonomy of Missingness	180
Model-Based Approaches to Analyses With Partial Missingness	181
Example 11.1: Multiple-Group Analyses Incorporating Missing Data	184

---

Example 11.2: Full Information Maximum Likelihood (FIML)	185
Extensions of the Multiple-Group Approach	
Example 11.3: Multiple Imputation of Missing Data	188
Summary	192
12. Latent Variable Framework for LGM Power Estimation	195
Power Estimation Within a Latent Variable Framework	196
Example 12.1: Power Estimation in LGM	198
Example 12.2: Power Estimation in a Multiple-Population Context	202
Example 12.3: Monte Carlo Power Estimation	206
Summary	208
13. Testing Interaction Effects in LGMs	213
Example 13.1: The Two-Factor Intercept-Slope Model	214
Summary	220
14. Summary	225
Advantages of LGM	226
Limitations of LGM	230
Concluding Remarks	231
References	233
Author Index	249
Subject Index	255
About the Authors	261

Latent Growth Curve Models. The common regression coefficient for job status on the GPA is estimates as  $\hat{\beta} = 0.12$  (s.e. 0.01), which is close to the multilevel regression estimates in Table 5.4. However, the model including all the job status variables does not fit well, with a chi-square of 202.1 (df=71,  $p < 0.001$ ) and an RMSEA of 0.10. We probably need many small modifications to make the model fit better. An advantage of latent curve analysis is that it can be used to analyze more complex structures. For instance, we may attempt to model the changes in hours spend on a job using a second latent curve model. The path diagram for the latent curve model for job status at the six time points is presented in Figure 14.6.

Latent Growth Curve Modeling With Categorical Variables Measurement Characteristics of the Ordered Categorical Variable Growth Modeling With Categorical Outcome Variables Software Implementation Example 10.1: LGM of Ordered Categorical Outcomes Summary 11. Missing Data Models A Taxonomy of Methods for Partial Missingness A Taxonomy of Missingness Model-Based Approaches to Analyses With Partial Missingness Example 11.1: Multiple-Group Analyses Incorporating Missing Data. TWO-FACTOR LGM FOR TWO TIME POINTS The simplest latent growth curve model involves one variable measured the same way at two time points. This book provides a comprehensive introduction to latent variable growth curve modeling (LGM) for analyzing repeated measures. It presents the statistical basis for LGM and its various methodological extensions, including a number of practical examples of its use. It is designed to take advantage of the reader's familiarity with analysis of variance and structural equation modeling (SEM) in introducing LGM techniques. Sample data, syntax, input and output, are provided for EQS, Amos, LISREL, and Mplus on the book's CD. Throughout the book, the authors present a variety of LGM techniques that