FLOWS, GENERALIZED DERIVATIVES, VARIATIONS AND NECESSARY CONDITIONS FOR AN OPTIMUM FOR CURVE MINIMIZATION PROBLEMS

H. SUSSMANN, Rutgers University.

ABSTRACT: Since the Pontryagin Maximum Principle was discovered in the 1950's, two competing approaches have been developed for proving extensions of the original result to theorems yielding stronger conclusions (for example, high-order conditions), or valid under weaker hypotheses (for example, right-hand sides that are Lipschitz rather than of class C^1) or both. The first, "primal," method, consists of working systematically with families of vector fields, making "needle variations," differentiating the corresponding endpoint map, and applying open mapping conditions based on some version of the Brouwer fixed point theorem. The second, "dual" approach, takes as its starting point the classical calculus of variations, and reduces the optimal control problems for vector fields and differential inclusions to problems of the more classical kind, by means of penalization methods in which violations of the constraints are prevented by addinga large penalty term in the cost functional. In the 1970's and 1980's, a lot of the progress in the non-smooth direction was made by means ofdual methods, while the primal approach yielded better results (with extra conclusions) for smooth systems but appeared not to lend itself to the study of non-smooth problems. This situation changed in the early 1990's, with the pioneering work of S. Lojasiewicz Jr., who showed how to get strong nonsmooth results with a primal technique. In the course, we will present a general primal methodology that has evolved out of Lojasiewicz's idea, and has led to a unified approach that applies to much larger classes of smooth and very non-smooth problems, including those that involve differential inclusions. This methodology is based on

(a) using vector fiels that are not necessarily continuous (following ideas of Alberto Bressan) in order to overcome the fact that the set-valued maps that occur in differential inclusion systems typically do not admit continuous selections, and to make sure that there are sufficiently many nice selections to make it possible to reduce all differential inclusion systems to systems of vector fields,

(b) relying systematically on regularity properties of flows rather than on properties of the vector fields that generate them, so as to exploit the fact that usually flows are more regular than their infinitesimal generators,

(c) considering abstract variations (of the kind studied by Krener, Knobloch, Bianchini, Stefani, Kawski, and others) rather than just the classical needle variations,

and, most importantly,

(d) using notions of derivative other than the classical one to differentiate the endpoint maps.

In particular, the course will give an axiomatic description of the notion of a "generalized differentiation theory" as a multifunctor between some appropriate categories, and provide a detailed presentation of one such theory—the "generalized differential quotients"—that is both fairly simple and of very broad applicability.
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A Papers by H. J. Sussmann

All these papers can be downloaded from H. Sussmann's Web page,

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http://www.math.rutgers.edu/ sussmann/currentpapers.html

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formulate the optimum nozzle problem in terms of one. independent variable.Â conditions at P, the Method of Characteristics may be used to solve for the flow conditions at R, Having the condiÂ tions at R and the wall shape defined by x, y, Gj, and a &t P the Method of Characteristics may be used to solve for the flow conditions at Q, It is apparent from the above discussion and Equation l4.Â partial derivatives of p and 8 an explicit expression for. the total derivative of p with respect to x at the point P. cannot be obtained in terms of conditions at P alone. Minimization The controllable system quantities are generator MW, controlled voltage magnitude, reactive power injection from reactive power sources and transformer tapping. The objective use herein is to minimize the power transmission loss function by optimizing the control variables within their limits.Â The OPF is capable of performing all of the control functions necessary for the power system.Â 3.1.6 Optimal Power Flow Challenges The demand for an OPF tool has been increasing to assess the.Â The Generalised Reduced Gradient is applied to the OPF problem [29] with the main motivation being the existence of the concept of the state and control variables, with load flow equations providing a nodal basis for the elimination of state variables. PDF | This paper presents the necessary and sufficient optimality conditions for problems of the fractional calculus of variations with a Lagrangian depending on the free end-points. The fractional derivatives are defined in the sense of Caputo.Â fractional variational problems, in terms of the Riemannâ€"Liouville and the Caputo derivative, are presented in [1, 2]. Here we develop further the theory by proving necessary optimality. conditions for more general problems of the fractional calculus of variations with a Lagrangian. that may also depend on the unspeciï¬ed end-points y(a), y(b). More precisely, the problem. under our study consists to minimize a functional which is deï¬ned in terms of the Caputo.