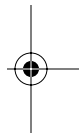
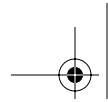


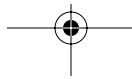
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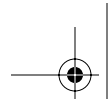




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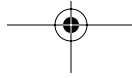


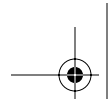
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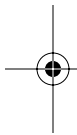




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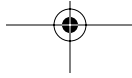
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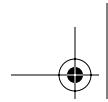
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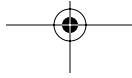
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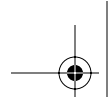
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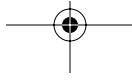
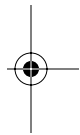
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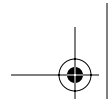
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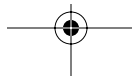
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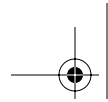
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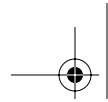
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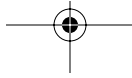
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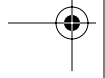
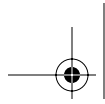
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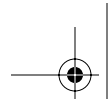
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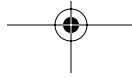


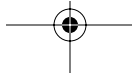
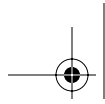


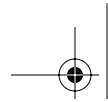


# Preface

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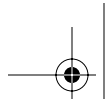
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Christian Menn  
[TO COME]

Frank J. Fabozzi, Ph.D., CFA, CPA is the Frederick Frank Adjunct Professor of Finance in the School of Management at Yale University. Prior to joining the Yale faculty, he was a Visiting Professor of Finance in the Sloan School at MIT. Professor Fabozzi is a Fellow of the International Center for Finance at Yale University and the editor of the *Journal of Portfolio Management*. He earned a doctorate in economics from the City University of New York in 1972. In 1994 he received an honorary doctorate of Humane Letters from Nova Southeastern University and in 2002 was inducted into the Fixed Income Analysts Society's Hall of Fame.





Many assets' return distributions are asymmetrical. In other words, the distribution is skewed to the left (or occasionally the right) of the mean (expected) value. In addition, most asset return distributions are more leptokurtic, or fatter tailed, than are normal distributions. The normal distribution has a kurtosis of 3.0, and Asset C has a fat tail with a kurtosis of 6. In the absence of skewness, Scenario 2 tests whether various levels of kurtosis affect asset allocations. Fat-Tailed and Skewed Asset Return Distributions examines this dilemma and offers readers a less technical look at how portfolio selection, risk management, and option pricing modeling should and can be undertaken when the assumption of a nonnormal distribution for asset returns is violated. Topics covered in this comprehensive book include an extensive discussion of probability distributions, estimating probability distributions, portfolio selection, alternative risk measures, and much more. Fat-Tailed and Skewed Asset Return Distributions provides a bridge between the highly technical theory